

in inventing situations to explain its absence. A revolutionary change in the fundamental concepts of physics, or a suitable modification of relativity theory, will solve the riddle in the future.

### References

- <sup>1</sup> v. Krzywoblocki, M. Z., "Time—dilatation dilemma and scale variation," *AIAA J.*, 2, 2213–2214 (1964).

## Reply by Author to V. S Ananthachar

M. Z. v. KRZYWOBLOCKI\*  
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A READER approaching the field of special relativity must keep in mind particularly two items: 1) no scientist ever stated or attempted to state that all of the processes in nature must obey the postulates of the Einstein special theory of relativity; 2) there exists more than one special theory of relativity. For more details concerning 1, the reader is referred to works by Rothman,<sup>1</sup> Schlegel,<sup>2</sup> and others. Concerning 2, a very profound theory of relativity of Whitehead<sup>3</sup> is seldom mentioned, and was applied to only a few cases. It may be of interest to acquaint oneself with the opinion of Whitehead on the subject of the theories of relativity. Whitehead wrote, "...Einstein, in my opinion, leaves the whole antecedent theory of measurement in confusion, when it is confronted with the actual conditions of our perceptual knowledge." In another place he says, "The possibility of other such laws, expressed in sets of differential equations other than Einstein's, arises from the fact that in my theory there is a relevant fact of nature which is absent on Einstein's theory." And, "The course of my argument has led me generally to couple my allusions to Einstein with some criticism.... But the worst homage we can pay to a genius is to accept uncritically formulations of truths which we owe to it (i.e., a method due to Einstein)." Recently, Schlegel<sup>2</sup> proposed his formulation of the theory of relativity, of Clausius and Lorentz (relativistic) processes. It is the expression of a deep intuition of a theoretical physicist who feels that the nature and the universe are not ruled autocritically by the geometrically oriented laws discovered by science in the past and that there are other laws which we do not know, as yet. It is known that some leading scientists in the field are a little disappointed with the purely geometrically oriented approach to model, and to describe, the laws of nature.† In this, Schlegel differs from both Whitehead and Einstein, and I do not hesitate to call his approach "Schlegel's formulation of the theory of relativity." Schlegel's formulation was nicely accepted. In the title of my note, published in 1964, I do not use the words "relativity, Einstein, etc." In the first two sections of it, I have presented a few general remarks. In the third section, entitled "Scale Variation in the Time-Dilatation Dilemma," the first sentence is "We accept the validity of the Schlegel hypothesis." In this section, there does not appear the name of Einstein even once. The contents of the section consist of the discussion of a particular example that illustrates Schlegel's idea (but not mine). I have shown in this example that by using Schlegel's formulation, it is possible to synchronize two clocks, one on the earth, another on an earth bound satellite, without the appearance of the so-called clock-paradox (should be rather the clock-ambiguity, as Schlegel has demonstrated). In this sec-

tion there are no remarks about cosmological aspects whatsoever; there are presented no new ideas, no new fundamentals. Referring to Ananthachar's remarks, I must state that I fail to see what he wants to say with reference to my note. He states that, "Assumption 1 is ruled out in relativity theory,..." It is not said in which relativity theory. Certainly not in the Schlegel theory, where this is one of the main assumptions. Hence this remark cannot be referred to my note. Ananthachar's Comment "A" refers to pre-Einstein theory. Comment "B" is very thoroughly explained in Schlegel's book, to which the reader is referred. Comment "C" refers to the kind of atomic clock; this point is not even mentioned by me. Comment "D" is discussed very thoroughly in Schlegel's book. Comment "E" refers to the situation when  $S_1$ , after some time, passes through  $S_2$ . Practically, this means that the spaceship will return to the earth and obviously the matter is closed. In Comment "F" Ananthachar states that my case assumes prepared journeys, as in the case of space travel. But I exactly discuss only the case of space travel. I do not discuss any case of journeys "...not necessarily planned by us..." (quoting Ananthachar). In the last paragraph, Ananthachar expresses his point of view; it is better that the paradox is left as it is. I absolutely never had and now have no objections to leaving the paradox in the Einstein theory of relativity as it is.

### References

- <sup>1</sup> Rothman, M. A., "Things that go faster than light," *Sci. Am.* 203, 142–150 (July 1960).  
<sup>2</sup> Schlegel, R., *Time and the Physical World* (Michigan State University Press, East Lansing, Mich., 1961).  
<sup>3</sup> Whitehead, A. N., *The Principle of Relativity* (Cambridge University Press, Cambridge, Mass., 1922).

## Comments on "Approximate Analytical Solution for Satellite Orbits Subjected to Small Thrust or Drag"

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IN a current paper by Brofman<sup>1</sup> the author has investigated the problems of satellites subjected to small thrust and drag. Both of these problems were previously studied by the writer.<sup>2–4</sup>

For the small tangential thrust problem, the expression of  $\theta$  presented in Ref. 2 can be greatly improved by combining Eqs. (3, 6, 23, and 39) in Ref. 2 and integrating the resulting expression as†

$$\theta = a\tau - a^2A \{ 3[X(\ln X - 1) + 1] + 3a[X(\ln X - 1)^2 + (X - 2)] + a^2[X(\ln X - 1)^3 + 3X(\ln X - 1) - (2X - 6)] \} \quad (1)$$

where the initial condition  $\tau = 0$  and  $\theta = 0$  has been employed, and

$$a \equiv \omega c/g_0 \quad \text{and} \quad X \equiv 1 - (\tau/A)$$

Equation (1) has been shown elsewhere<sup>5</sup> and its accuracy against numerical integration was depicted in Ref. 5. With  $\theta$  shown in Eq. (1) instead of Eq. (37) in Ref. 2, it is thought

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† The recent theory of R. Dicke from Princeton demonstrates very clearly that the geometry in the theory of relativity is completely unnecessary. A scalar and a tensor fields are completely sufficient.

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† See Nomenclature list in Ref. 2.

that the results presented in Ref. 2 have the accuracy comparable to that shown in the current paper.

For the drag problem, the variations of the motion from the average were brought out in Ref. 4; therefore the statement about the drag problem in the introduction of the current paper should be amended.

### References

<sup>1</sup> Brofman, W., "Approximate analytical solution for satellite orbits subjected small thrust or drag," AIAA J. 5, 1121-1128 (1967).

<sup>2</sup> Zee, C. H., "Low constant tangential thrust spiral trajectories," AIAA J. 1, 1581-1583 (1963).

<sup>3</sup> Zee, C. H., "Low thrust oscillatory spiral trajectory," *Astronaut. Acta* IX, 201-207 (1963).

<sup>4</sup> Zee, C. H., "Trajectories of satellites under the influence of air drag," *AIAA Progress in Astronautics and Aeronautics: Celestial Mechanics and Astrodynamics* edited by V. G. Szebehely (Academic Press Inc., New York, 1964) Vol. 14, pp. 101-112.

<sup>5</sup> Cohen, M. J., "Low-thrust spiral trajectory of a satellite of variable mass," AIAA J. 3, 1946-1949 (1965).

## Comment on "One-Dimensional Minimum-Time Rendezvous for a Thrust-Limited Rocket"

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REFERENCE 1 considers optimum methods to deliver a given final mass to a specified position-velocity state in minimum time. Two cases are compared, the specified final mass case and the specified initial mass case. The main result for mass ratio  $\mu$  in the case of initial specified mass is given by Eq. (18) in the reference as

$$z_{1f} = \mu - 2\mu e^{-(z_{2f} + \ln \mu)/2} - z_{2f} + 1$$

where  $z_{1f}$  and  $z_{2f}$  are normalized position and velocity variables. The equation is described as "transcendental" and no solution is exhibited for  $\mu$ . Although this equation is indeed transcendental in  $z_{2f}$  it is algebraic in terms of  $\mu$  and may be solved explicitly to yield

$$\mu^{1/2} = e^{-z_{2f}/2} + [e^{-z_{2f}} + (z_{1f}^{1/2} + z_{2f} - 1)]^{1/2}$$

### Reference

<sup>1</sup> Anderson, G. M., Falb, P. L., and Robinson, A. C., "One-dimensional minimum-time rendezvous for a thrust-limited rocket," AIAA J. 5, 1017-1019 (1967).

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## Errata: "Langmuir Probe Diagnosis of Turbulent Plasmas"

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[AIAA J. 4, 451-459 (1966)]

MESSRS. T. W. Johnston and A. K. Ghosh of RCA Victor Company, Ltd. have kindly pointed out minor numerical errors in certain formulas of the reference cited above. The last term of the second bracket of Eq. (28) should read  $2(\Delta T/\bar{T})(\Delta \bar{\eta})$  instead of  $(\Delta T/\bar{T})(\Delta \bar{\eta})$ . Equation (21) should read:

$$\begin{aligned} J/j_0 = & 1 - \frac{1}{8}(1 - 3\bar{\eta})(1 + \bar{\eta})^{-1}(\Delta T/\bar{T})^2 + \\ & \frac{1}{2}(1 - \bar{\eta})(1 + \bar{\eta})^{-1}(\Delta n/\bar{n})(\Delta T/\bar{T}) + \\ & (1 + \bar{\eta})^{-1}[(\Delta n/\bar{n})(\Delta \bar{\eta}) - \frac{1}{2}(\Delta T/\bar{T})(\Delta \bar{\eta})] \end{aligned}$$

and Eq. (31) should read

$$\begin{aligned} (\Delta j)^2/j_0^2 = & (\Delta n/\bar{n})^2 + \frac{1}{4}(1 - \bar{\eta})^2(1 + \bar{\eta})^{-2}(\Delta T/\bar{T})^2 + \\ & (1 + \bar{\eta})^{-2}(\Delta \bar{\eta})^2 + (1 - \bar{\eta})(1 + \bar{\eta})^{-1}(\Delta n/\bar{n})(\Delta T/\bar{T}) + \\ & 2(1 + \bar{\eta})^{-1}(\Delta n/\bar{n})(\Delta \bar{\eta}) + (1 - \bar{\eta})(1 + \bar{\eta})^{-2}(\Delta T/\bar{T})(\Delta \bar{\eta}) \end{aligned}$$

In the limit  $\bar{\eta} \gg 1$  these two equations will yield correct forms of Eqs. (22) and (32).

Received August 1, 1967.

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## Erratum: "Transformation of Hypersonic Turbulent Boundary Layers to Incompressible Form"

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[AIAA J. 5, 1202-1203 (1967)]

ON page 1202, last term of equation for  $\mu_s \sigma / \bar{\mu}$  should read  $K(\bar{C}_f/2)(\zeta_f^2/3)$ .

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